Modelling of Miniaturized Coplanar Striplines Based on $YBa_2Cu_3O_{7-x}$ Thin Films

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***Abstract* – A miniaturized $YBa_2Cu_3O_{7-x}$ coplanar stripline (CPS) structure is investigated field–theoretically by means of a partial wave synthesis [1,2,3]. The high–T_c superconductor material is described by the two–fluid model and the London theory [4]. Approximation formula for effective permittivity, attenuation, and characteristic impedance are deduced from the numerical results.**

With high–T_c superconducting coplanar stripline structures miniaturized transmission lines with very small attenuation and with nearly no dispersion can be realized. In contrast to microstrip lines [5] no via holes are necessary and the miniaturization is not limited by the substrate thickness to avoid undesirable coupling between neighbour strips. In comparison to the coplanar waveguide (CPW) structure the CPS structure has the additional advantage that no parasitic modes are excited.

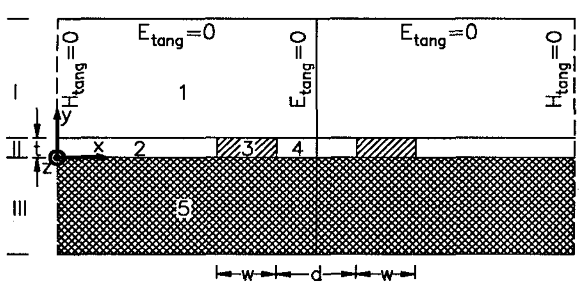

Fig. 1. Cross section of the CPS structure.

The field calculation, which our results are based on, is performed by means of a partial wave synthesis which takes into consideration the exact field distribution also within the superconducting regions. Fig. 1 shows the cross section of the considered CPS structure. For analysis by partial wave synthesis the structure is surrounded by electric and magnetic walls. The fundamental mode of the CPS structure, which we are interested in, is an odd mode. So we can introduce an electric wall in the symmetry plane and then we have to consider only one half of the structure, which is subdivided into 3 layers and 5 regions. In each of the regions the electromagnetic field is expanded into a series of partial waves consisting of a combination of electric (LSE_x) and magnetic (LSH_x) x longitudinal section waves, which are described by the following Helmholtz equations.

$$\Delta\Pi_x - j\omega\mu\kappa\Pi_x = 0 \qquad (LSE_x \text{ waves})$$
$$\Delta\tilde{\Pi}_x - j\omega\mu\kappa\tilde{\Pi}_x = 0 \qquad (LSH_x \text{ waves}) \qquad (1)$$

where κ is given by

$$\kappa = \begin{cases} j\omega\epsilon + \sigma_d & \text{in regions 1,2,4, and 5} \\ \sigma_s = \sigma_n - j\cdot\dfrac{1}{\omega\mu_0\lambda_l^2} & \text{in region 3} \end{cases} \qquad (2)$$

and σ_n is the normal conductivity and λ_l is the London penetration depth of the superconductor.

The field components are then given by eq. (3).

$$\begin{aligned} E_x &= \frac{\partial^2\Pi_x}{\partial x^2} - j\omega\mu\kappa\Pi_x \\ H_x &= \frac{\partial^2\tilde{\Pi}_x}{\partial x^2} - j\omega\mu\kappa\tilde{\Pi}_x \\ E_y &= \frac{\partial^2\Pi_x}{\partial x\partial y} - j\omega\mu\frac{\partial\tilde{\Pi}_x}{\partial z} \\ H_y &= \kappa\frac{\partial\Pi_x}{\partial z} + \frac{\partial^2\tilde{\Pi}_x}{\partial x\partial y} \\ E_z &= \frac{\partial^2\Pi_x}{\partial x\partial z} + j\omega\mu\frac{\partial\tilde{\Pi}_x}{\partial y} \\ H_z &= -\kappa\frac{\partial\Pi_x}{\partial y} + \frac{\partial^2\tilde{\Pi}_x}{\partial x\partial z} \end{aligned} \qquad (3)$$

The following onset for propagation in positive z direction

$$\begin{aligned} \Pi_x &= \sum_m \left\{ \left[A_m\cdot e^{jk_{xm}x} + B_m\cdot e^{-jk_{xm}x} \right]\cdot \left[C_m\cdot e^{jk_{ym}y} + D_m\cdot e^{-jk_{ym}y} \right] \right\}\cdot e^{j(\omega t - k_z z)} \\ \tilde{\Pi}_x &= \sum_m \left\{ \left[\tilde{A}_m\cdot e^{j\tilde{k}_{xm}x} + \tilde{B}_m\cdot e^{-j\tilde{k}_{xm}x} \right]\cdot \left[\tilde{C}_m\cdot e^{j\tilde{k}_{ym}y} + \tilde{D}_m\cdot e^{-j\tilde{k}_{ym}y} \right] \right\}\cdot e^{j(\omega t - k_z z)} \end{aligned} \qquad (4)$$

leads to common expressions for the field components.

The next step is the establishment of partial waves in each layer. This is easily accomplished in the homogenious layers I and III. Fulfilling the boundary conditions in x direction the wavenumber k_{xm} for each partial wave m is

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determined. In layer II which consists of the three regions 2, 3 and 4, k_{xm} can only be determined after matching the tangential electric and magnetic fields at the boundaries between these regions. After determination of m_0 partial waves in each layer we obtain a system of equations while matching the tangential field components at the boundaries between the different layers by the moment method. As test functions separable terms describing the dependence of the field components of each partial wave in x direction are used. The resulting system of equations is linear in the coefficients of the partial waves but non-linear in the wavenumber k_z of the propagating mode, implicitly included in the matrix elements. To get a non-trivial solution the determinant of the linear homogenious system must be zero and fulfilling this condition k_z can be calculated by means of a variational method.

The following calculations are based on a $YBa_2Cu_3O_{7-x}$ film with a London penetration depth of 240 nm at 77 K, a normal conductivity of $4 \cdot 10^6$ S/m [6] and a thickness of t = 250 nm on a lanthan aluminate substrate with a relative permittivity of $\epsilon_r = 24$. Fig. 2 and 3 show the effective relative permittivity ϵ_{reff}, fig. 4 and 5 the attenuation α, and fig. 6 and 7 the characteristic impedance Z_w of the CPS waveguide versus conductor distance d at a conductor width of w = 5 μm and versus conductor width w at a conductor distance of d = 5 μm, respectively. Fig. 8 shows the attenuation versus frequency for a conductor distance of d = 2 μm and a conductor width of w = 5 μm.

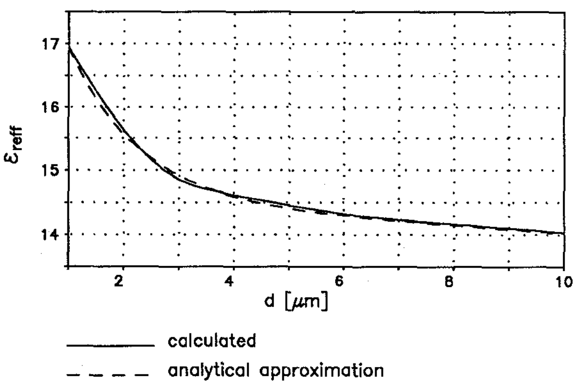

Fig. 2. Effective permittivity of CPS vs. conductor distance.

In order to obtain analytical approximation formula for the effective permittivity ϵ_{reff}, the attenuation α, and the characteristic impedance Z_w of the above described miniaturized CPS structure in dependence of the conductor distance d, the conductor width w, and the frequency f, the results of the partial wave synthesis are used. In the first step the dependencies on d, w, and f are treated separately. Because the miniaturized superconducting CPS structure has nearly no dispersion in the considered frequency range up to 100 GHz, the dependency of ϵ_{reff} and Z_w on frequency is neglected.

For the analytical approximation of the d and w dependencies the functions

$$f_i(x) = c_{1i} + c_{2i} \cdot \exp(c_{3i} \cdot x) + c_{4i} \cdot x \qquad (5)$$

are used and for the f dependency of α the function

$$g(x) = c_5 \cdot x^{c_6} \qquad (6)$$

is used.

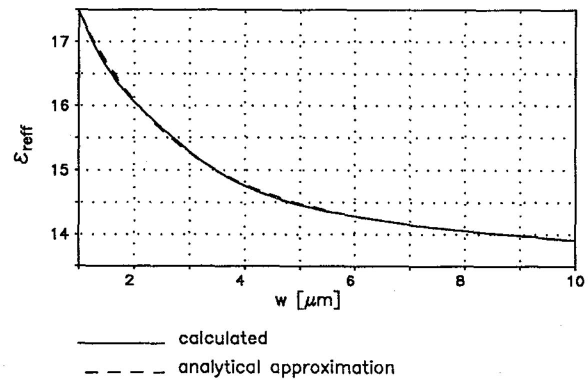

Fig. 3. Effective permittivity of CPS vs. conductor width.

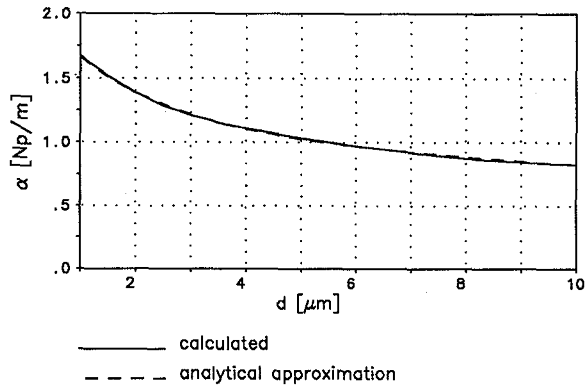

Fig. 4. Attenuation of CPS vs. conductor distance.

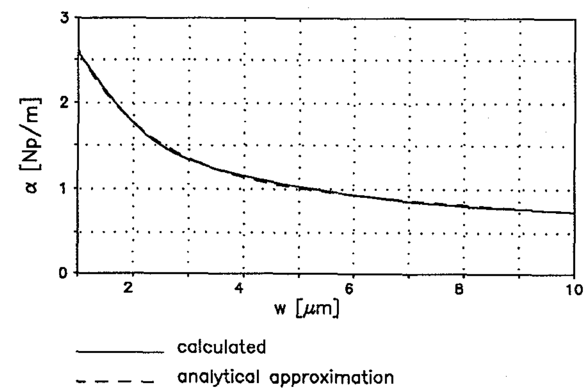

Fig. 5. Attenuation of CPS vs. conductor width.

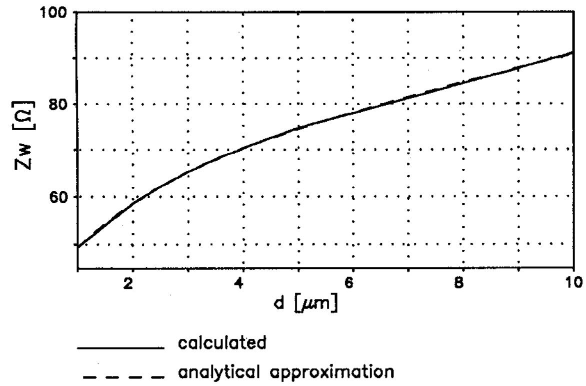

Fig. 6. Characteristic impedance of CPS vs. conductor distance.

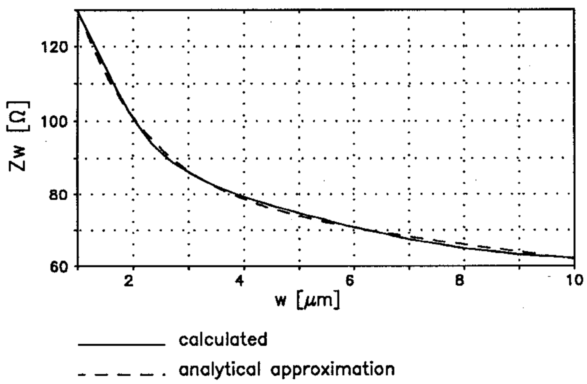

Fig. 7. Characteristic impedance of CPS vs. conductor width.

Minimizing the relative quadratic deviation from the numerical results we obtain

$$\epsilon_{reff1}(d) = 14.6 + 5.36 \cdot \exp(-0.8\ \mu m^{-1} \cdot d) - 0.058\ \mu m^{-1} \cdot d \quad \text{at } w = 5\mu m \tag{7}$$

$$\epsilon_{reff2}(w) = 14.4 + 5.45 \cdot \exp(-0.56\ \mu m^{-1} \cdot w) - 0.051\ \mu m^{-1} \cdot w \quad \text{at } d = 5\mu m \tag{8}$$

$$\alpha_1(d) = 1.08\ Npm^{-1} + 1.03\ Npm^{-1} \cdot \exp(-0.52\ \mu m^{-1} \cdot d) - 0.026\ Npm^{-1} \cdot d \quad \text{at } w = 5\ \mu m \text{ and } f = 10\ GHz \tag{9}$$

$$\alpha_2(w) = 1.15\ Npm^{-1} + 3.2\ Npm^{-1} \cdot \exp(-0.76\ \mu m^{-1} \cdot w) - 0.042\ Npm^{-1} \mu m^{-1} \cdot w \quad \text{at } d = 5\ \mu m \text{ and } f = 10\ GHz \tag{10}$$

$$\alpha_3(f) = 0.0142\ Npm^{-1} \cdot (f \cdot GHz^{-1})^{1.992} \quad \text{at } d = 2\ \mu m \text{ and } w = 5\ \mu m \tag{11}$$

$$Z_{w1}(d) = 60.9\Omega - 25.5\Omega \cdot \exp(-0.56 \mu m^{-1} \cdot d) + 3.02\Omega \mu m^{-1} \cdot d \quad \text{at } w = 5\ \mu m \tag{12}$$

$$Z_{w2}(w) = 81\ \Omega + 107\ \Omega \cdot \exp(-0.75\ \mu m^{-1} \cdot w) - 1.9\Omega \mu m^{-1} \cdot w \quad \text{at } d = 5\ \mu m \tag{13}$$

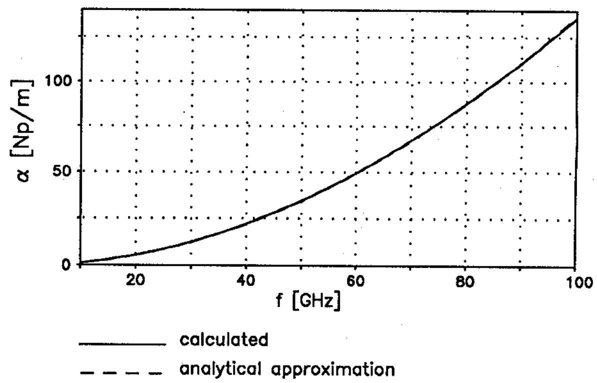

Fig. 8. Attenuation of CPS vs. frequency.

The dashed lines in Fig. 2 through 5 represent these analytical approximations. In a second step we synthesize the expressions of eq. (7),(8) and (12),(13) and obtain as a multipliable superposition for ϵ_{reff} and Z_w:

$$\epsilon_{reff}(d,w) = \epsilon_{reff1}(d) \cdot \frac{\epsilon_{reff2}(w)}{\epsilon_{reff2}(5\mu m)} \tag{14}$$

$$Z_w(d,w) = Z_{w1}(d) \cdot \frac{Z_{w2}(w)}{Z_{w2}(5\mu m)} \tag{15}$$

For the attenuation we find the average of an additive and a multipliable superposition of the conductor distance and conductor width dependencies and a multipliable superposition of the frequency dependency to match best.

$$\alpha(d,w,f) = 0.5 \cdot \left[\alpha_1(d) + \alpha_2(w) - \alpha_2(5\mu m) + \alpha_1(d) \cdot \frac{\alpha_2(w)}{\alpha_2(5\mu m)}\right] \cdot \frac{\alpha_3(f)}{\alpha_3(10GHz)} \tag{16}$$

In the considered parameter range of $d = 1...10\ \mu m$, $w = 1...10\ \mu m$, and $f = 10...100$ GHz the maximum aviation from the numerical results is 1% in effective permittivity, 3% in attenuation, and 4% in characteristic impedance.

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